Year 9 Higher

Plotting Quadratic Graphs:

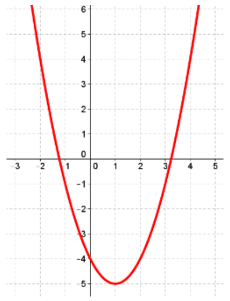
$y = x^2 - 2x - 4$

When $x = -2, y = (-2)^2 - (2 \times -2) - 4 = 4$

x	-2	-1	0	1	2	3	4
y	4	-1	-4	-5	-4	-1	4

Coordinates are (-2, 4), (0,-4) etc.

Plot these coordinates on a coordinate grid and plot a **SMOOTH** curve.



Upper and Lower Bounds:

15 (Nearest Integer)

Lower Bound = 14.5

Upper Bound = 15.5

$\mathbf{14.5 \le 15 < 15.5}$

20.9 (3sf)

LB = 20.85 and UB = 20.95

$\mathbf{20.85 \le 20.9 < 20.95}$

Averages from Grouped Frequency Tables:

Height, h (cm)	Freq	Midpoint, m	$m \times$ Freq.
$0 < h \le 10$	15	5	$5 \times 15 = 75$
$10 < h \le 20$	37	15	$15 \times 37 = 555$
$20 < h \le 30$	26	25	$25 \times 26 = 650$
$30 < h \le 40$	22	35	$35 \times 22 = 770$
Total	**100**		**2050**

Estimate for the Mean = $\frac{2050}{100} = 20.5$cm

Using midpoints gives us an estimate as exact values are unkown

Modal Class = $10 < h \le 20$ **(The category with the biggest frequency!)**

Class in which the Median lies: The median is the $\left(\frac{n+1}{2}\right)^{th}$ Value. There are 20 people, so the median is the $\left(\frac{100+1}{2}\right)^{th} = 55.5^{th}$ Value. The median is therefore in the $20 < h \le 30$ category!

Multiplying and Dividing in Standard Form:

$(4.2 \times 10^3) \times (3 \times 10^4) = (4.2 \times 3) \times (10^3 \times 10^4)$

$= 12.6 \times 10^7$

But our answer is not in Standard Form. We need to write it as: $\mathbf{1.26 \times 10^8}$

$(7.5 \times 10^9) \div (2.5 \times 10^6) = (7.5 \div 2.5) \times (10^9 \div 10^6)$

$= \mathbf{3 \times 10^3}$

AND/OR Rules

Independent: 2 events that do not affect each outcome

Mutually Exclusive: 2 events that cannot happen at the same time

For Independent Events: $P(A \text{ and } B) = P(A) \times P(B)$

For Mutually Exclusive Events: $P(A \text{ or } B) = P(A) + P(B)$

Simple Interest:

£2000 is paid into an account that pays 5% simple interest per annum (pa). The amount in the account after 3 years is: $£2000 + (2000 \times 0.05 \times 3) = £2300$

Solving Linear Equations:

Linear Equations can have fractional and negative solutions!

$18 - 7x = 3(2x - 8)$

Expand the brackets

$18 - 7x = 6x - 24$

Add $7x$ from both sides as it is the smallest

$(+7x) \qquad (+7x)$

$18 = 13x - 24$

$(+24) \qquad (+24)$

$42 = 13x$

$(\div 13) \qquad (\div 13)$

Solution: $x = \frac{42}{13}$

$\frac{3x+8}{2} = 1$

$(\times 2) \qquad (\times 2)$

$3x + 8 = 2$

$(-8) \qquad (-8)$

$3x = -6$

$(\div 3) \qquad (\div 3)$

Solution: $x = -2$

$\frac{5x-3}{4} = \frac{2x+9}{3}$

Multiply both sides by 12 as it is the LCM of 4 and 3

$\frac{12(5x-3)}{4} = \frac{12(2x+9)}{3}$

12÷4=3 and 12÷3=4

$3(5x - 3) = 4(2x + 9)$

Expand the brackets

$15x - 9 = 8x + 36$

$(-8x) \qquad (-8x)$

$7x - 9 = 36$

$(+9) \qquad (+9)$

$7x = 45$

$(\div 7) \qquad (\div 7)$

Solution: $x = \frac{45}{7}$

Remember to simplify your fractions if you can!

Expand and Simplify:

$(3x - 7)(5x - 2)$

$= 15x^2 - 6x - 35x + 14$

$= \mathbf{15x^2 - 41x + 14}$

$(2x + 9)^2 = (2x + 9)(2x + 9)$

$= 4x^2 + 18x + 18x + 81$

$= \mathbf{4x^2 + 36x + 81}$

$(5x + 7)(5x - 7)$

$= 25x^2 - 35x + 35x - 49$

$= \mathbf{25x^2 - 49}$

This is an example of DOTS (Difference of Two Squares)

Negative and Fractional Indices

$x^{-n} = \frac{1}{x^n}$

$x^{\frac{1}{n}} = \sqrt[n]{x}$

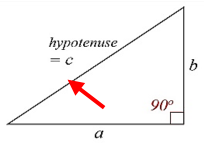
$6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

$\left(\frac{4}{7}\right)^{-2} = \left(\frac{7}{4}\right)^2 = \frac{49}{16}$

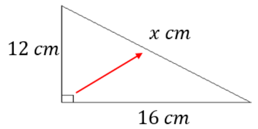
$121^{\frac{1}{2}} = \sqrt{121} = 11$

$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

Pythagoras' Theorem:



$c^2 = a^2 + b^2$



$a^2 + b^2 = c^2$

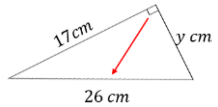
$12^2 + 16^2 = x^2$

$144 + 256 = x^2$

$x^2 = 400$

$(\sqrt{}) \qquad (\sqrt{})$

$\mathbf{x = 20cm}$



$a^2 + b^2 = c^2$

$y^2 + 17^2 = 26^2$

$y^2 + 289 = 676$

$(-289) \qquad (-289)$

$y^2 = 387$

$(\sqrt{}) \qquad (\sqrt{})$

$y = \sqrt{387}$ **cm or** $\mathbf{y = 19.7cm(3sf)}$

Compound Measures:

Speed (m/s, km/h, mph)= $\frac{Distance}{Time}$

Pressure (N/m)= $\frac{Force}{Area}$

Density (kg/m³, g/cm³) = $\frac{Mass}{Volume}$

Solving Quadratics by factorising:

$x^2 - x - 42 = 0$

We require 2 numbers that add to make the coefficient of x (-1) and multiply to make the constant term (-42). The two numbers are -7 and 6. We then factorise the quadratic:

$(x - 7)(x - 6) = 0$

Either: $x - 7 = 0$ or $x + 6 = 0$

$(+7) \quad (+7) \quad (-6) \quad (-6)$

Solutions: $\mathbf{x = 7 \text{ or } x = -6}$

Compound Interest:

£2000 is paid into an account that pays 4.8% compound interest per annum (pa). The amount in the account after 3 years is:

$£2000 \times 1.048^3 = £2302.05(2dp)$

Reverse Percentages:

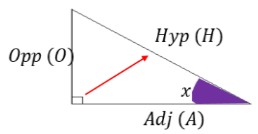
A Football shirt is reduced by 17%. It now costs £51.66. The original cost was:

$\mathbf{51.46 \div 0.83 = £62}$

A House increases in price by 16%. It is now worth £162,400. The original price was:

$\mathbf{162400 \div 1.16 = £140,000}$

Trigonometry:

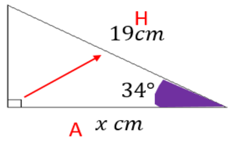


$\sin x = \frac{O}{H}$

$\cos x = \frac{A}{H}$

$\tan x = \frac{O}{A}$

Use the word **SOHCAHTOA** to help you remember!



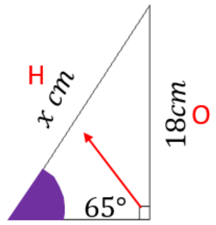
$\cos(34) = \frac{A}{H}$

$\cos(34) = \frac{x}{19}$

$(\times 13) \qquad (\times 13)$

$x = 19 \times \cos(34)$

$\mathbf{x = 15.8cm(3sf)}$



$\sin(65) = \frac{O}{H}$

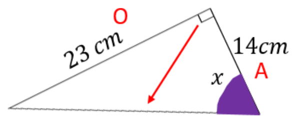
$\sin(65) = \frac{18}{x}$

$(\times x) \qquad (\times x)$

$x \times \sin(65) = 18$

$(\div \sin(65)) \qquad (\div \sin(65))$

$x = \frac{18}{\sin(65)} = \mathbf{19.9cm(3sf)}$



$\tan(x) = \frac{O}{A}$

$\tan(x) = \frac{23}{14}$

$(\tan^{-1}) \qquad (\tan^{-1})$

$x = \tan^{-1}\left(\frac{23}{14}\right)$

$\mathbf{x = 58.7°(3sf)}$

Factorise:

$-10x - 35 = -5(2x + 7$

$4x^2 + \frac{3}{2}x = \frac{1}{2}x(8x + 3)$

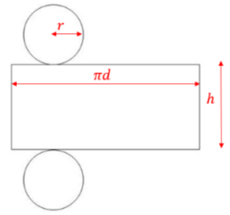
You can also take out negatives and fractions as factors!

$Arc\ Length = \frac{\theta}{360} \times \pi d$

$Area\ of\ a\ Sector = \frac{\theta}{360} \times \pi r^2$

Volume and Surface Area:

Total Area of each face.



For a Cylinder:

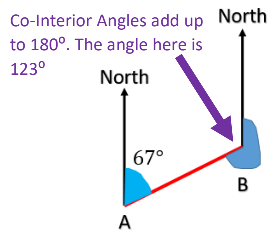
$\mathbf{S = 2\pi r^2 + \pi dh}$

$\mathbf{V = \pi r^2 h}$

Bearings:

- 3 Figures
- Measure from North (000°)
- Measure Clockwise

Co-Interior Angles add up to 180°. The angle here is 123°



The bearing of B from A is 067°. The bearing of A from B is 247°

Inverse Proportion

3 Pipes take 60mins to water a field. 1 Pipe will take 180mins to water the same field. Therefore, 10pipes will take 18mins

Sequences

Find the first 3 terms of the sequence with nth term: $3n^2 - 7$

$n = 1, \Rightarrow (3 \times 1^2) - 7 = -4$

$n = 2, \Rightarrow (3 \times 2^2) - 7 = 5$

$n = 3, \Rightarrow (3 \times 3^2) - 7 = 20$

Fnd the first 3 terms of the sequence given by: $n(n-4)$

Remember: $n(n-4) = n \times (n-4)$

$n = 1, \Rightarrow 1 \times (1 - 4) = 1 \times -3 = -3$

$n = 2, \Rightarrow 2 \times (2 - 4) = 2 \times -2 = -4$

$n = 3, \Rightarrow 3 \times (3 - 4) = 3 \times -1 = -3$

Congruency: Two Shapes are congruent if they are the same size after a translation, reflection or rotation.

Similarity: Two shapes are similar if one is a perfect enlargement of the other

SolvingSimulatenous Equations using Elimination

$4x + 7y = 15 \qquad (1)$

$5x - 2y = 8 \qquad (2)$

Make the coefficient of x or y the same to eliminate one of the vaiables

$(1) \times 2 \Rightarrow 8x + 14y = 30$

$(2) \times 7 \Rightarrow 35x - 14y = 56$

Add the two equations together as the signs of y are **different**

$43x = 86$

$(\div 43) \qquad (\div 43)$

$x = 2$

To find our y value, we need to substitute x=2 into either equation. Using equation 1:

$(4 \times 2) + 7y = 15$

$8 + 7y = 15$

$(-8) \qquad (-8)$

$7y = 7$

$(\div 7) \qquad (\div 7)$

$y = 1$

Solution: $\mathbf{x = 2, y = 1}$

$3x + 5y = 14 \qquad (1)$

$7x + 2y = 23 \qquad (2)$

Make the coefficient of x or y the same to eliminate one of the vaiables

$(1) \times 7 \Rightarrow 21x + 35y = 98$

$(2) \times 3 \Rightarrow 21x + 6y = 69$

Subtract the two equations together as the signs of x are **the same**

$29y = 29$

$(\div 29) \qquad (\div 29)$

$y = 1$

To find our x value, we need to substitute $y = 1$ into either equation. Using equation 2:

$7x + (2 \times 1) = 23$

$7x + 2 = 23$

$(-2) \qquad (-2)$

$7x = 21$

$(\div 7) \qquad (\div 7)$

$x = 3$

Solution: $\mathbf{x = 3, y = 1}$

Solving Linear Inequalities:

Use the balancing method!

$18 - 7x < 6x - 8$

Add $7x$ from both sides as it is the smallest

$(+7x) \qquad (+7x)$

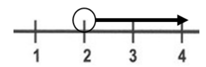
$18 < 13x - 8$

$(+8) \qquad (+8)$

$26 < 13x$

$(\div 13) \qquad (\div 13)$

Solution: $\mathbf{x > 2}$



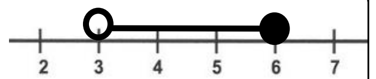
$2 < \frac{x}{3} + 1 \le 3$

$(-1) \quad (-1) \quad (-1)$

$1 < \frac{x}{3} \le 2$

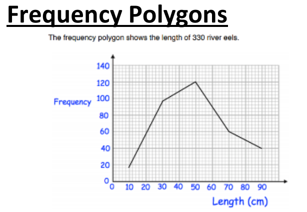
$(\times 3) \quad (\times 3) \quad (\times 3)$

Solution: $\mathbf{3 < x \le 6}$



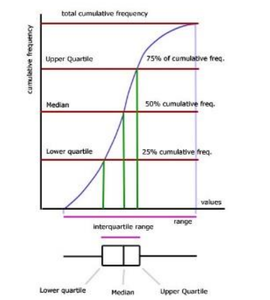
Integers that satisfy this inequality are: 4, 5, 6

Frequency Polygons



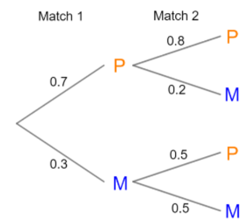
Plot the frequency against the midpoints of each class

Cumulative Frequency and Box Plots



Plot the cumulative frequency against the upper limit of

Tree Diagrams



P(Peter wins both Games)

$= 0.7 \times 0.8 = 0.56$